Quantum Algorithms I

The Quantum Computing Stack

5	Applications
3	Algorithms
2	Circuits
4	Hardware
1	Mechanics

You are Here!

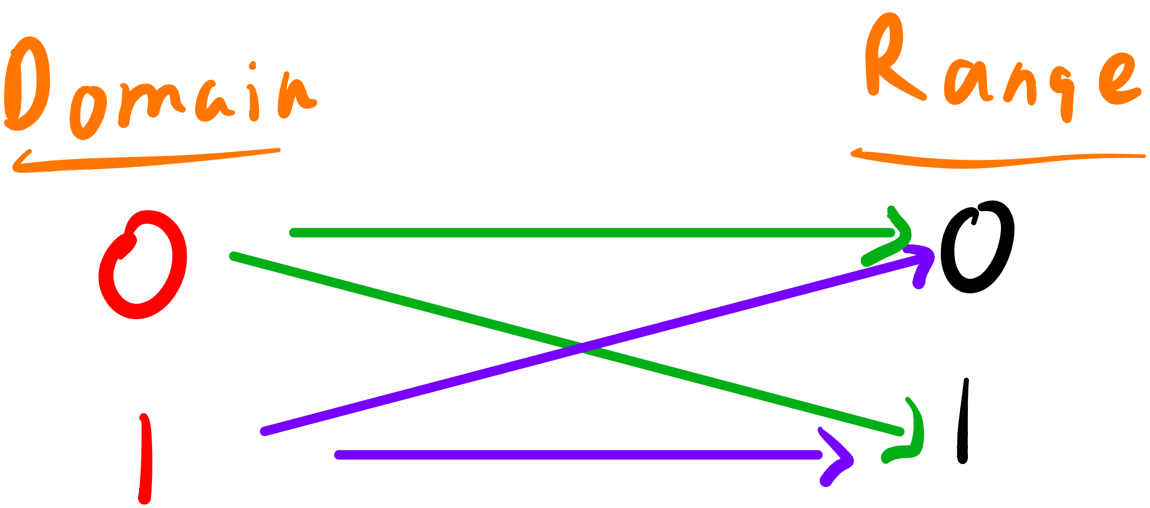
Algorithms

Speedup

- Deutsch's Not useful exponential
- Grover's useful Quadratic
- Shor's Useful exponential
- VQE (maybe)

Contrived Problem

Difficult to compute $f(x)$



$f(0) = 0$ or 1

$f(1) = 0$ or 1

Want to know

Does $f(0) = f(1)$?

evaluate evaluate

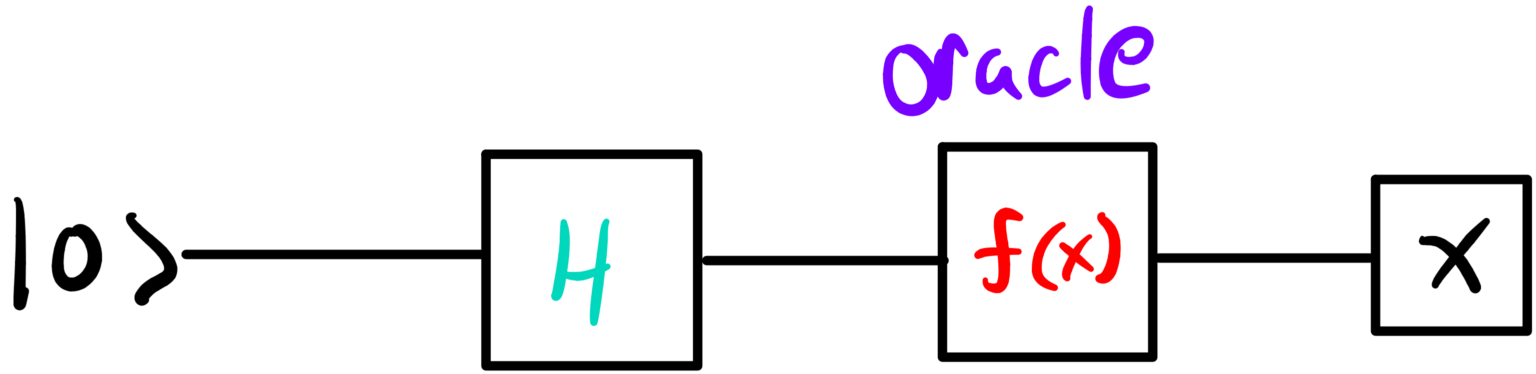
Classical Approach

2 evaluations

Deutch's Algorithm

1 evaluation

Naïve Quantum Approach



$$|+\rangle \quad f(|+\rangle)$$

Probablistic → Deterministic

Superposition

Phase-Kickback

Deutch's Algorithm

Superposition

Hadamard

$$H|0\rangle = |+\rangle$$

Same Sign

$$|+\rangle = \frac{1}{\sqrt{2}}\left(+|0\rangle + |1\rangle\right)$$

50% 50%

$$H|+\rangle = |0\rangle$$

$$H|1\rangle = |-\rangle$$

Different Sign

phase

$$|-\rangle = \frac{1}{\sqrt{2}}\left(+|0\rangle - |1\rangle\right)$$

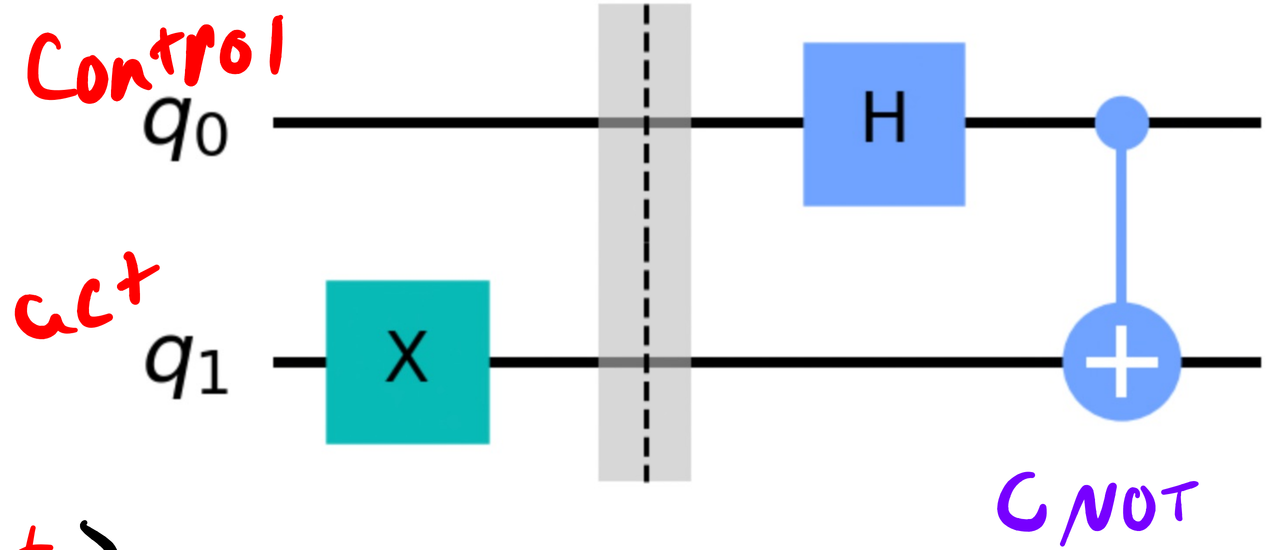
$$H|-\rangle = |1\rangle$$

Entanglement

- How we Analyze qubits
- Outcomes of Measurement

<u>Seperability</u>

Entangled



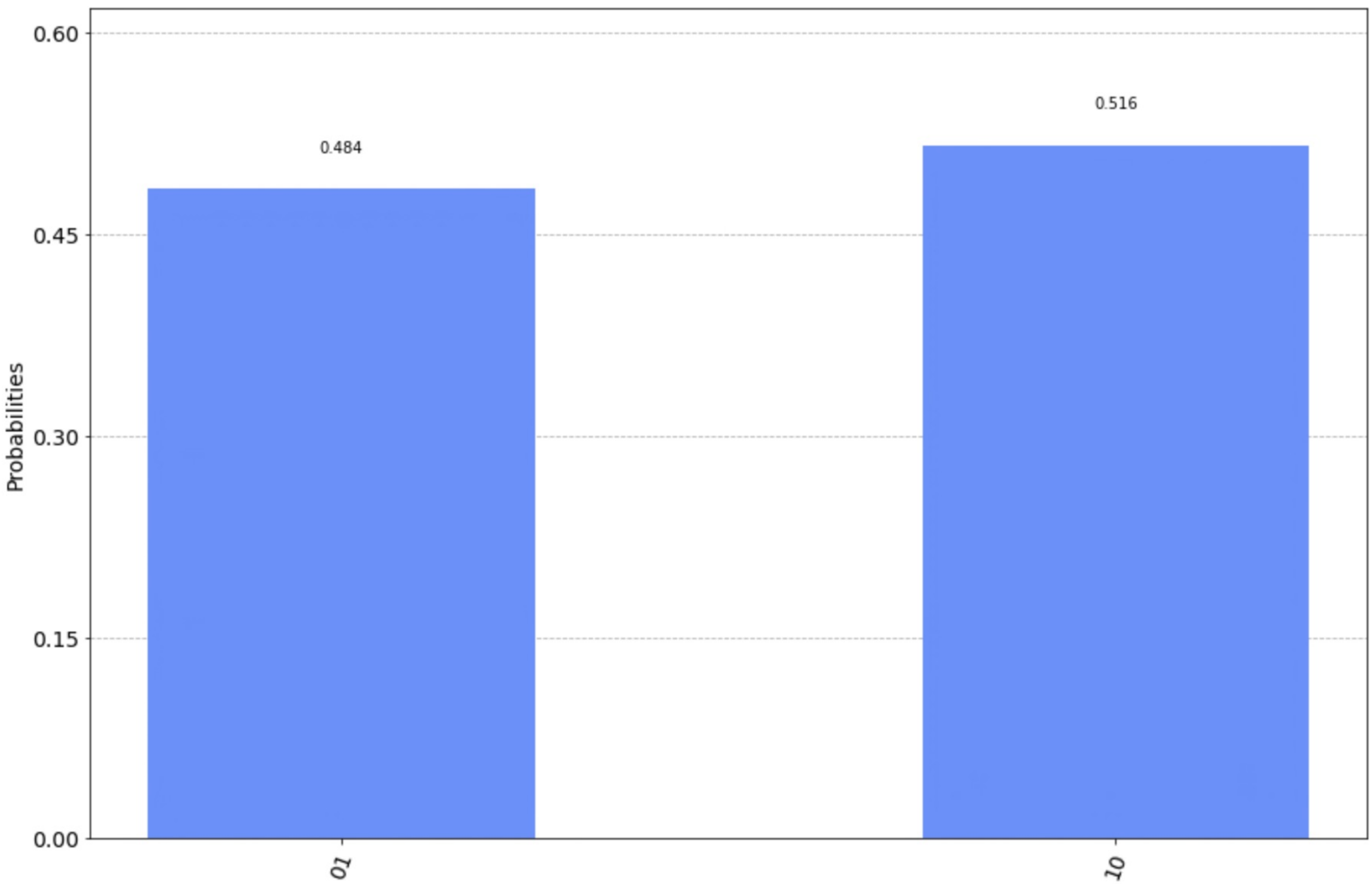
$$|q_0\rangle = H|0\rangle = |+\rangle$$

$$|q_1\rangle = X|0\rangle = |1\rangle$$

$$|q_1 q_0\rangle = |1+\rangle = |1\rangle \otimes \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) = \frac{|10\rangle + |11\rangle}{\sqrt{2}}$$

$$\text{CNOT}|1+\rangle = \frac{|10\rangle + |01\rangle}{\sqrt{2}}$$

$$|q_1 q_0\rangle = \frac{|10\rangle + |01\rangle}{\sqrt{2}}$$



What is Seperable?

$$\boxed{|q_1\rangle = a|0\rangle + b|1\rangle} \qquad \boxed{|q_0\rangle = c|0\rangle + d|1\rangle}$$

$$|q_1 q_0\rangle = \left(a|0\rangle + b|1\rangle\right) \otimes \left(c|0\rangle + d|1\rangle\right)$$

$$= ac|00\rangle + \underline{ad}|01\rangle + \underline{bc}|10\rangle + bd|11\rangle$$

Can you find a set of values a, b, c, d That satisfy

CNOT from earlier

$$|q_1 q_0\rangle = |10\rangle + |01\rangle$$

$$ac = 0$$
$$ad = 1$$
$$bc = 1$$
$$bd = 0$$

Entangled

$\Updownarrow$

Inseperable

Importance of Phase

$$|+\rangle = \frac{1}{\sqrt{2}}\left(\overset{1/2}{|0\rangle} + \overset{1/2}{|1\rangle}\right)$$

$$|-\rangle = \frac{1}{\sqrt{2}}\left(\overset{1/2}{|0\rangle} - \overset{1/2}{|1\rangle}\right)$$

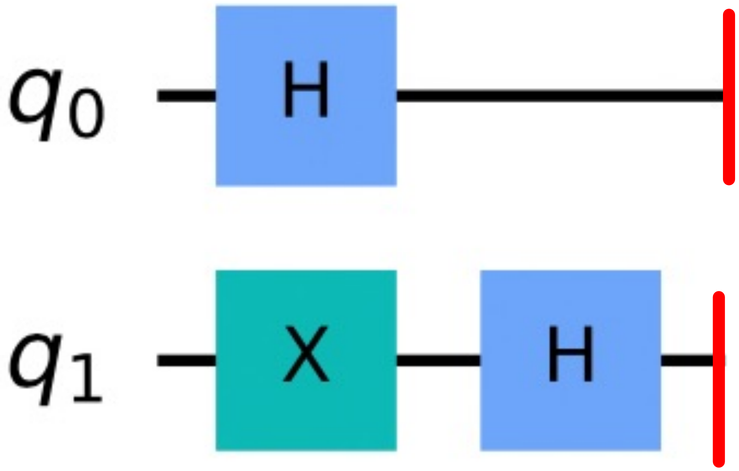
Same result when measured

$$H|+\rangle = |0\rangle$$

$$H|-\rangle = |1\rangle$$

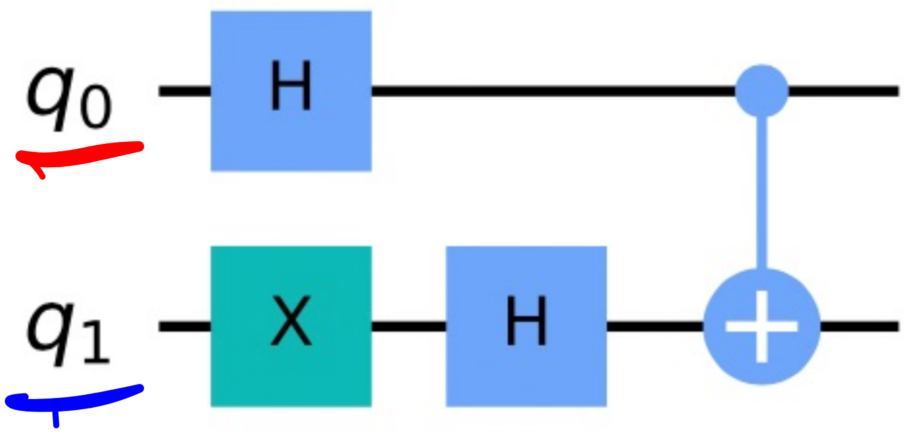
Different result when operated on

Phase Kickback



what is q_0 and q_1 after these gates?

Kicking Phase Back!



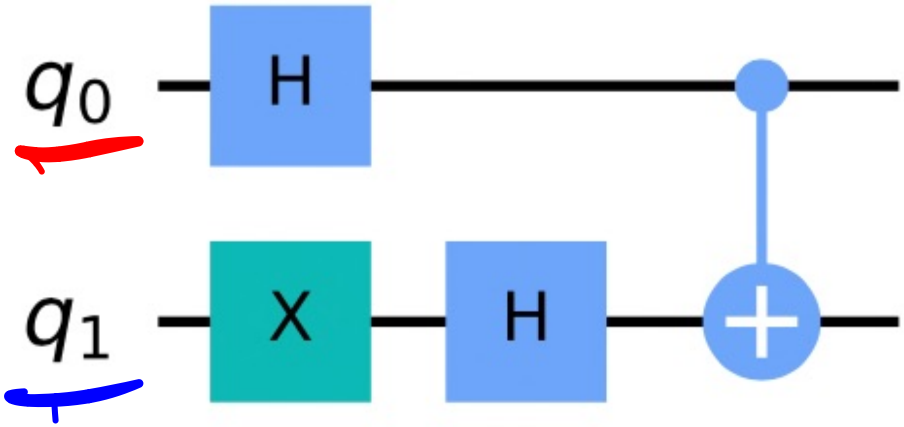
$$|q_1 q_0\rangle = |-+\rangle$$

$$|-+\rangle = |-\rangle \otimes |+\rangle$$

$$= \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) \otimes \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$= \frac{1}{2}\left(|01\rangle + |01\rangle - |10\rangle - |11\rangle\right)$$

Kicking Phase Back!



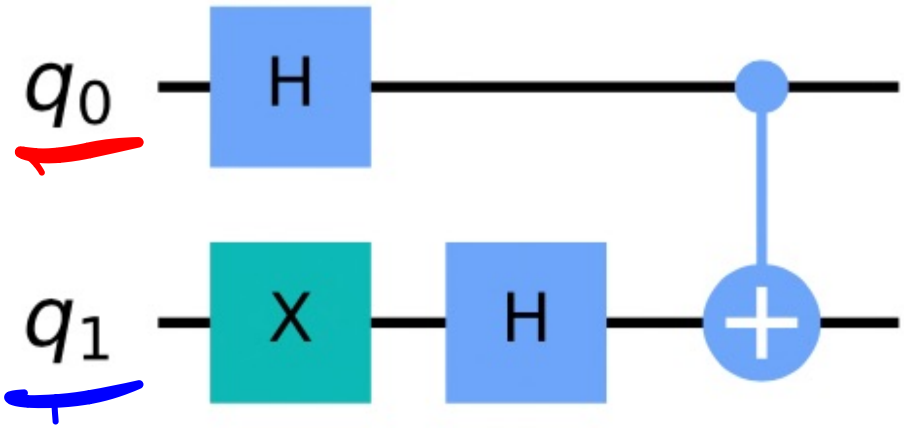
$$|q_1 q_0\rangle = |-+\rangle$$

$$|-+\rangle = \frac{1}{2}\left(|01\rangle + |01\rangle - |10\rangle - |11\rangle\right)$$

$$\text{CNOT}\,|-+\rangle =$$

Key word: Parasite

Control Changed?



$$|q_1 q_0\rangle = |-+\rangle$$

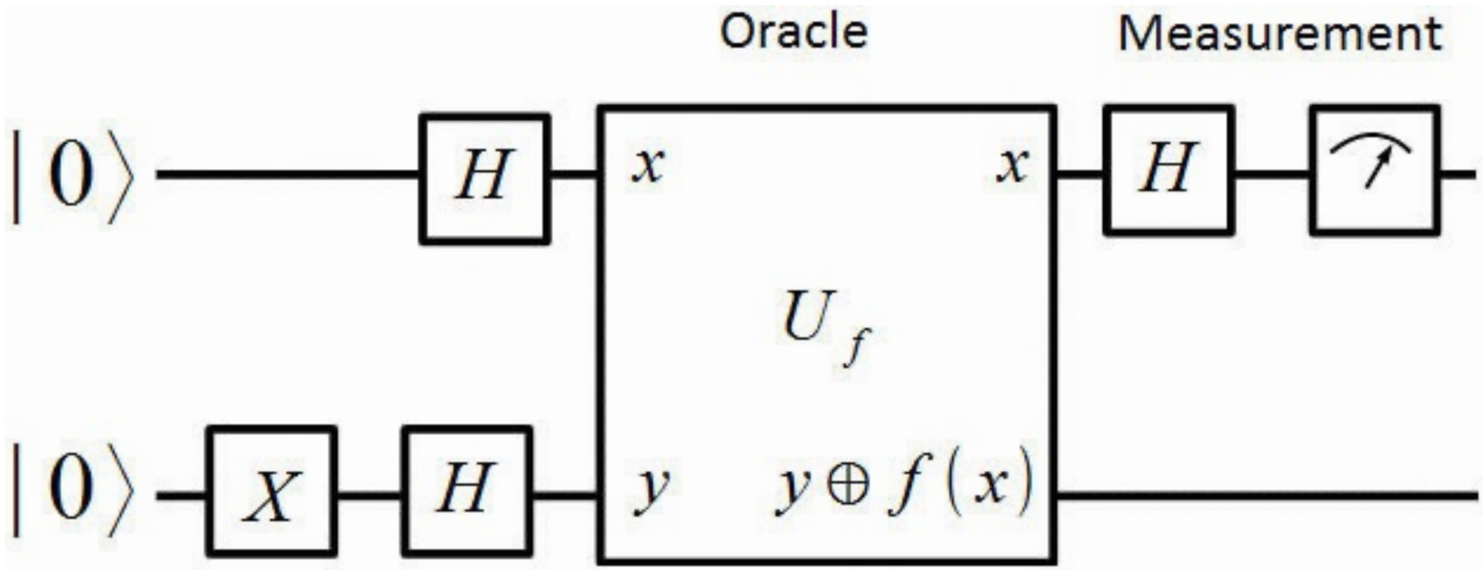
$$\text{CNOT}|-+\rangle = |--\rangle$$

Control Phase changes

Action State changes

Confusing!

Deutch's Algorithm



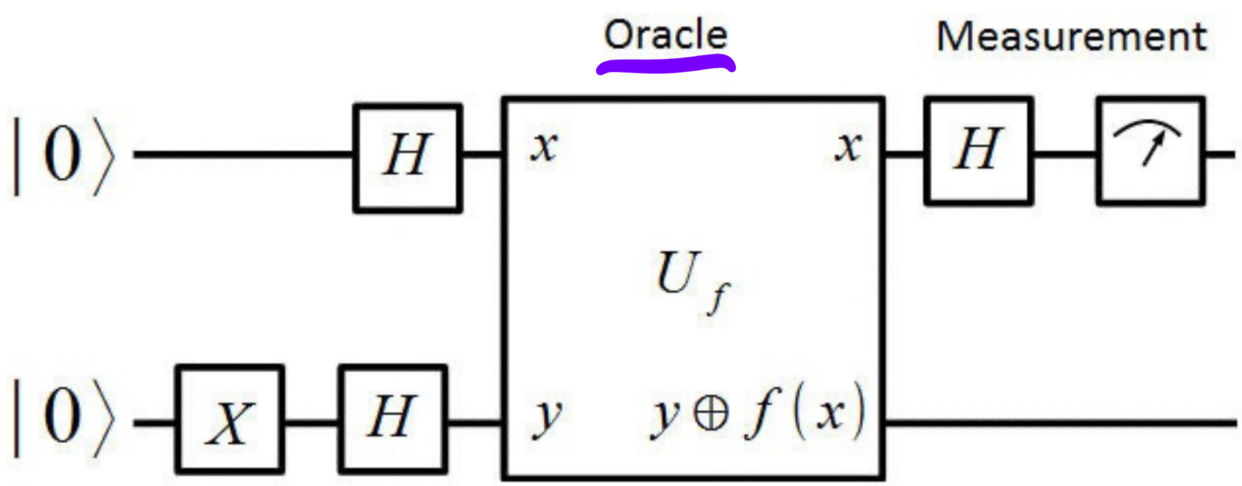
4. Measure

1. Super-position

2. Phase Kickback Result

3. Phase -> State

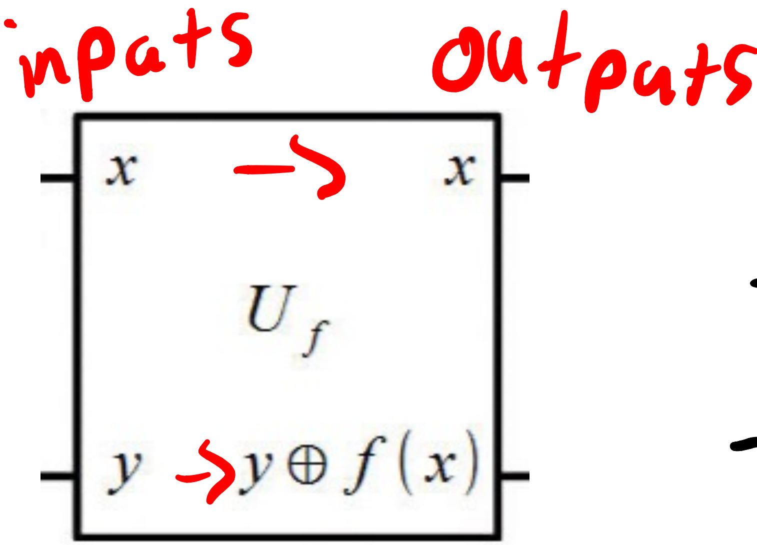
Deutch's Algorithm



In just 1 call to f using our oracle, we find if

$$f(0) = f(1) \text{ or } f(0) \neq f(1)$$

Oracle



- Blackbox
- Generalization
- Only care about input and output

0,1 0,1

$$U_f|y\rangle|x\rangle = |y \oplus f(x)\rangle|x\rangle$$

similar to CNOT

$$CNOT|y\rangle|x\rangle = |y \oplus x\rangle|x\rangle$$